Name: PD: Date:

Quiz 3 – Power, Product, and Quotient Rules (I Can D*Rive!)

1) Let $y = x^6 + \frac{2}{x}$. Find y'''. The question asks us to find the third derivative. We want to use the power rule, so the first thing we'll do is to rewrite the **second** term in exponential form. $y = x^6 + 2x^{-1}$

$y' = 6x^5 + (-2x^{-2})$; $y'' = 30x^4 + (+4x^{-3})$; $y''' = 120x^3 + (-12x^{-4})$

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2) Find the **point(s)** on the curve $f(x) = 3x^2 + 4x - 3$ where the tangent line is horizontal.

Where is the tangent line horizontal? When the slope is 0! We also know that the slope of the tangent line is what the first derivative tells us. So, here's our work: $f'(x) = 6x + 4 = 0 \Rightarrow x = -2/3$. That's the *x* value at which this happens at least. To find the **point** at which this happens I'll also need the *y* value. $f(-2/3) = 4/3 - 8/3 - 3 = -13/3$.

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3) Find $f'(x)$ when $f(x) = (x^2 - 2x)(4x^3 + 7x - 3)$.

Straight product rule:
$$f(x) = (x^2 - 2x)(4x^3 + 7x - 3)$$
$$f'(x) = (2x - 2)(4x^3 + 7x - 3) + (x^2 - 2x)(12x^2 + 7)$$

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4) If $f(3) = -2,\ f'(3) = -1,\ g(3) = 3,$ and $g'(3) = 0$, find $F'(3)$ *if* $F(x) = 2f(x)g(x)$.

$F'(x) = (2f'(x))g(x) + 2f(x)g'(x)$. We just use the product rule with $2f(x)$ as the first function. Once I have the derivative of *F*(*x*), all I need do is substitute the 3 in. So …

$F'(3) = (2f'(3))g(3) + 2f(3)g'(3) = (2(-1))(3) + 2(-2)(0) = -6$

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5) Find $\frac{dy}{dx}$ when $y = \frac{\sqrt{x}}{x-1}$.

Quotient rule! $y' = \dfrac{\left(\frac{1}{2\sqrt{x}}\right)(x-1) - \sqrt{x}(1)}{(x-1)^2}$. And writing $\sqrt{x}$ as $x^{-1/2}$ is also fine.

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6) In question (4), you were told that $f(3) = -2$ and $f'(3) = -1$. Write the equation of the line tangent to the curve defined by the function *f*, at the point where $x = 3$.

For equation of tangent line we need: **point:** $(3, -2)$ and **slope:** $m = f'(3) = -1$. So … equation of line is $(y - y_0) = m(x - x_0) \Rightarrow (y - (-2)) = (-1)(x - 3)$. And you're done.

The difference between stupidity and genius is that genius has its limits. - Einstein

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7) Let $y=\left(\frac{1}{\sqrt{x}}+x\right)\left(x^2-3x\right)$. Find $\frac{d^2y}{dx^2}$ by using derivative rules – do not multiply y out.

Product rules! $y=\left(\frac{1}{\sqrt{x}}+x\right)\left(x^2-3x\right)$; $y'=\left(-\frac{1}{2}x^{-3/2}+1\right)\left(x^2-3x\right)+\left(\frac{1}{\sqrt{x}}+x\right)(2x-3)$. But we want the second derivative. That's going to use the product rule **twice**.

$$y'=\left(-\frac{1}{2}x^{-3/2}+1\right)\left(x^2-3x\right)+\left(\frac{1}{\sqrt{x}}+x\right)(2x-3)$$

$$y''=\left\{\left(\frac{3}{4}x^{-5/2}\right)\left(x^2-3x\right)+\left(-\frac{1}{2}x^{-3/2}+1\right)(2x-3)\right\}+\left\{\left(-\frac{1}{2}x^{-3/2}+1\right)(2x-3)+\left(\frac{1}{\sqrt{x}}+x\right)(2)\right\}$$

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8) Find the derivative of $\frac{x^7-14x^2+4}{x^3-2x^2+4}$.

Quotient rule! $y'=\frac{\left(7x^6-28x\right)\left(x^3-2x^2+4\right)-\left(x^7-14x^2+4\right)\left(3x^2-4x\right)}{\left(x^3-2x^2+4\right)^2}$